Preface

This note developed based on the course Advanced Geometry by Professor Manolis at ShanghaiTech University is a first step towards Algebraic Geometry, a subject that is known to be elusive, abstract, and inaccessible. Manolis brings it to undergraduate life in a digestive and approachable way.

Disclaimer. While the lectures are made psychologically comfortable, which I definitely agree that is one of the most important factors forming a course, this note is not. There are English syntax errors, symbolic inconsistencies, and even wrong proofs in the note. The readers should be brave enough to find and attack them as exercises, although this note can be improved in many ways.

Motivation. The main motivation of writing notes for this course is as follows. While David A Cox et al. [3] introduce algebraic geometry at undergraduate level from a computational perspective and the target students of Math 145[1] by Ravi Vakil are from mathematical discipline, there is as far as I know no course or book that takes algebraic geometry to engineering students, in a proof-based manner. If a book is indeed desired, this course note is the very first step.

Story. But who will take it? There is always the same hesitation after each lecture that whether should I devote myself to developing this note. I have papers to read, code to write, girls to date with, and video games to play. It is always the case that I write after each lecture, with the aroma of coffee permeating in the windy afternoon or evening, tired yet happy. Mathematics is like an ocean, where the one diving deeply witnesses the beauty. I find a way of diving: hacking proofs, giving examples (not waiting for examples), and designing exercises. The note grows like that I am pouring the water into an apertured bucket. The more I wrote, the more errors I made. Will the constant dripping from the bucket hollow the stone? It was until the end of the course I did not realize the answer. Hesitation does not be turned into regret.

Content.

Part I: Point Set Topology. The topology part follows strictly from Mendelson [1], of which 80% are covered, including open set and closed set, continuity, compactness, connectedness, etc..

Part II: Algebraic Geometry. In the second part of the course, some fundamental theorems and results in algebraic geometry are considered. To develop them,

[1] http://math.stanford.edu/~vakil/17-145/

however, some just enough prerequisites in (commutative) algebra are introduced beforehand. This reverse engineering approach is used, not due to the habit of a mathematical hacker, for inevitable reasons. This is the first time to teach such a course in ShanghaiTech, before which no one knows what should be taught, and the target students major in engineering, who do not learn any topology or abstract algebra before. The main results developed in this part include Hibert's Nullstellensatz, Noether Normalization, and some dimension theory.

References

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